*Termo-estatística 1/2014*

***Noturno***

**Exercícios sugeridos**

**4.** Consider the experiment shown in Figure 5. Assume that the distribution function is of the form

$$f(x) = \frac{CL^2}{(x^2 + L^2)^{3/2}}$$

where $C$ is a constant and $L$ is the distance between the screen and the gas container.

(a) Determine the constant $C$. [Hint: $L^2dx/(x^2 + L^2)^{3/2} = d(x/(x^2 + L^2)^{1/2})$.]

(b) If $L = 2m$, what is the probability that an atom will strike the screen between the points $x = -1$ cm and $+1$ cm?

(c) What is the probability that an atom will strike in the region $x \leq 1$ m?

(d) If a total of $10^6$ atoms strike the screen, what is the probable number of atoms that would be found in the region $dx$ of $x = 3$ m? At the point $x = -1$ m?

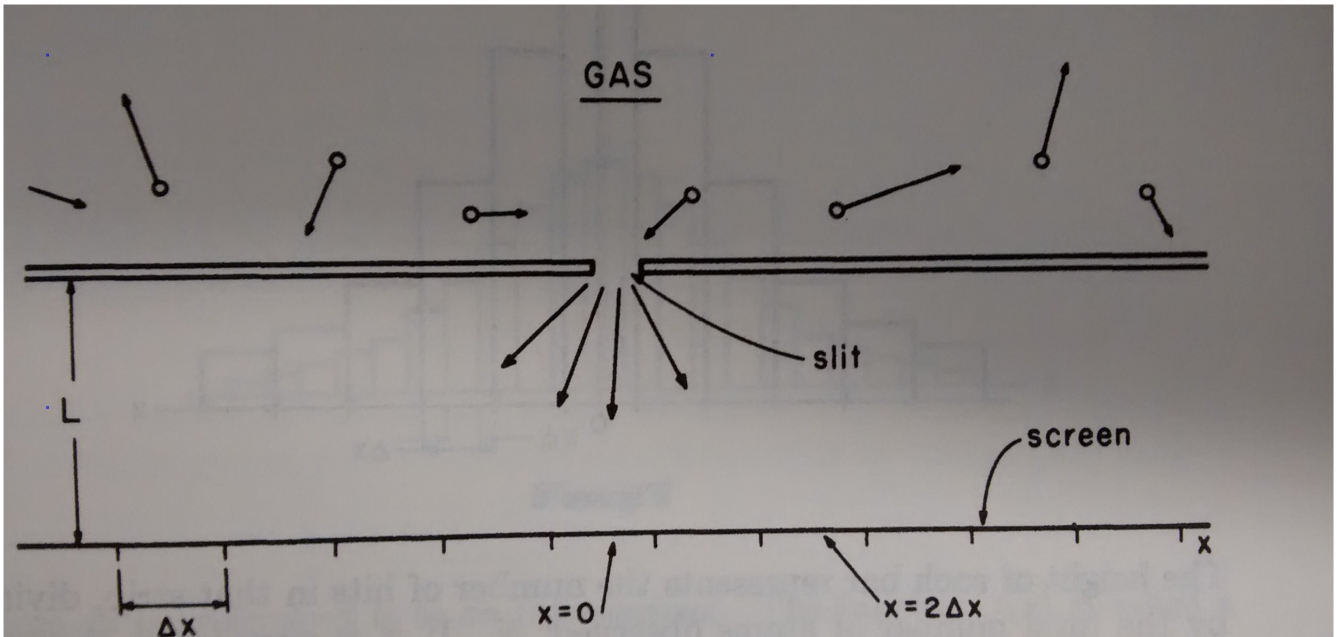


**Figure 5**

$x = -1$ m?

5. In the manufacturing of an electronic instrument, it is found that the probability $P(n)$ that an instrument has $n$ defects within six months is

$$P(0) = 0.1, \quad P(1) = 0.4, \quad P(2) = 0.25,$$
$$P(3) = 0.15, \quad P(4) = 0.08, \quad P(5) = 0.02$$

(a) What is the average number of defects of all the instruments in the first six months?

(b) If you bought an instrument, what is the most likely number of defects it would have in the first six months?

(c) If the manufacturer must repair the instrument, which of the numbers — (a) or (b) — is significant in determining his expense?

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6. The speed $s$ of cars on a road is found to be given by the distribution function

$$f(s) = As \exp\left(\frac{-s}{s_0}\right) \qquad (0 \leq s \leq \infty)$$

where $A$ and $s_0$ are constants.

(a) Determine $A$ in terms of $s_0$.

(b) A radar unit can differentiate only between speeds that differ by small amounts $\Delta s$. In the region of what speed $s_m$ is it most likely to find a particular car?

(c) What is the probability of the radar unit's actually finding a car in this region?

(d) What is the average speed of the cars?

(e) Assume that the number of accidents that a car has is proportional to its speed — say, $bs$ per month (where $b$ is some constant). What is the average number of accidents per month on this road, assuming that $N$ cars use it?

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7. A man who enjoys throwing darts at a vertical pole placed against a wall finds from long experience that the probability of a dart's hitting the wall at a distance $x$ from the center of the pole (where $x = 0$) is well represented by the distribution function

$$f(x) = \frac{\lambda}{\sqrt{\pi}} e^{-\lambda^2(x-x_0)^2} \qquad (-\infty \leq x \leq \infty)$$

where $\lambda$ and $x_0$ both depend on the day of the week.

(a) What is the average distance $\bar{x}$ by which he misses the target in terms of $\lambda$ and $x_0$?

(b) What is the dispersion of his shots?

(c) If he throws 200 darts, what is the probable *number* of darts that hit between $x_1$ and $x_1 + dx_1$?

(d) He finds that $\lambda$ and $x_0$ vary with the days as follows:

| | $\lambda$ (*inches*)$^{-1}$ | $x_0$ (*inches*) |
|---|---|---|
| Monday | $\frac{1}{2}$ | 2.0 |
| Wednesday | $\frac{1}{3}$ | −1.0 |
| Friday | $\frac{1}{4}$ | 0.0 |

On what day does he hit most consistently near his average position?

(e) On what day is he most likely to hit the target? If the target is 0.1 inch wide, what is the probability (approximately)?

9. Consider the distribution function for the velocity of a particle $f(\mathbf{v}) = (c/\pi)^{3/2} e^{-c\mathbf{v}^2}$, where $c$ is a constant and each velocity component can range from $-\infty$ to $+\infty$.

(a) Using the results of Appendix A, show that $f(\mathbf{v})$ is normalized to unity. Note that this involves three integrals that can be evaluated separately.

(b) Determine the probability that $v_x \geq 0$, $v_y \geq 0$, and $v_z \leq 0$ (i.e., simultaneously). Explain its dependency on the constant $c$.

(c) Obtain an expression for the probability that the $x$ component of $\mathbf{v}$ is in the range $dv_x$ of $v_x$, regardless of the values of $v_y$ and $v_z$. Explain your reasoning [along the lines of Equation (21)].

**10.** An interesting example of a bimodal distribution function is provided by Old Faithful, the famous geyser in Yellowstone National Park. If we let $t$ represent the time between the eruptions of the geyser (in minutes), then the probability $f(t)dt$ that the next eruption occurs in an interval $dt$ of $t$ can be reasonably represented by the sum of two Gaussians

$$f(t) = 0.34\sqrt{\frac{\beta_1}{\pi}}\, e^{-\beta_1(t-51)^2} + 0.66\sqrt{\frac{\beta_2}{\pi}}\, e^{-\beta_2(t-74)^2}$$

where $\beta_1 = 0.021$ and $\beta_2 = 0.017$.

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(a) Determine to the nearest minute the time when $f(t)$ is a maximum [that is, the two modes of $f(t)$]. Which is the most probable value of $f(t)$? Note that when one Gaussian is a maximum, the other Gaussian is essentially zero.

(b) What is the value of $\bar{t}$? To obtain this, integrate over $-\infty \leq t \leq \infty$. The result is essentially the same as integrating over $0 \leq t \leq \infty$ because $f(t) \simeq 0$ for negative values of $t$.

(c) Using the same range of integration as in (b), determine the standard deviation, $\sigma(t)$ [by way of comparison, if $f(t) = (\sqrt{\beta_2/\pi})e^{-\beta_2(t-74)^2}$ then $\sigma(t) \simeq 5.4$.] What percentage of $\bar{t}$ is $\sigma(t)$? Is Old Faithful faithful?

. Consider the three distribution